

Computer Modelling of Structures

Comparison of Designing Flat Slabs using Finite Element Analysis with Traditional Methods



Course; DT024/4

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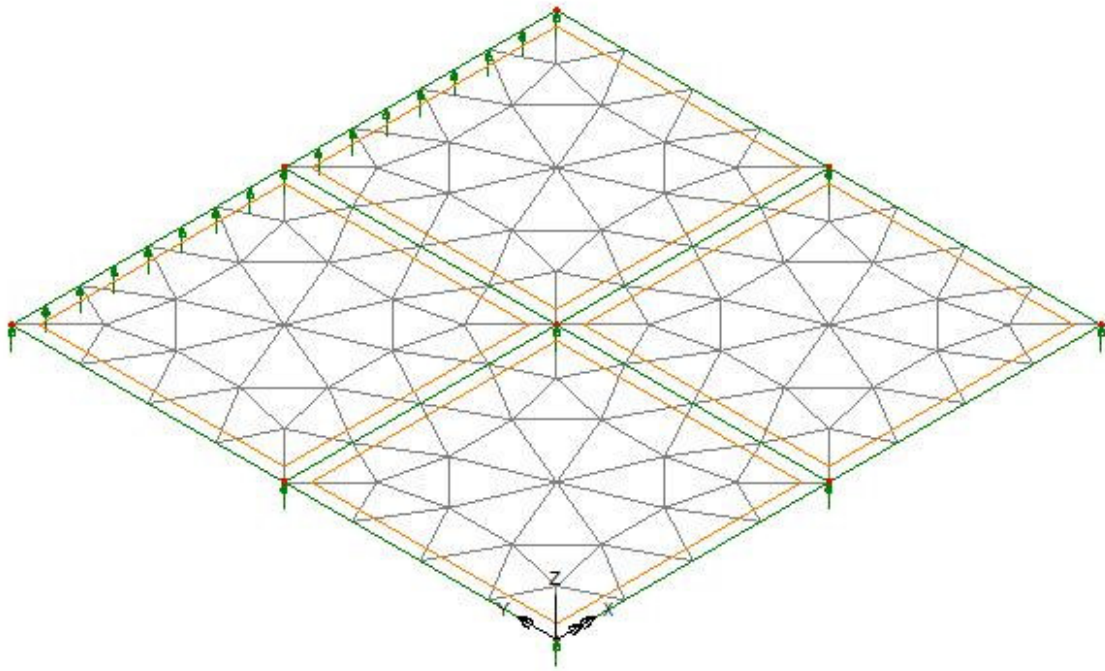
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Flat Slab to be designed

Panel	6m x6m
Fcu	40 N/mm ²
Fy	460 N/mm ²
Cover	20 mm
Depth	200 mm
Max bar diameter	12 mm
Conc. Wt.	24 KN/M ³
UDL Dead	4.8 KN/M ²
UDL Live	5 KN/M ²

BS8110

A flat slab floor is a reinforced concrete slab supported directly by concrete columns without the use of beams.

The flat floor slab has many advantages over the beam and slab floor. The simplified formwork and overall reduced storey height make it more economical. Windows can be extended up to the underside of the slab and there are no beams to obstruct the light.

The analysis of a flat slab structure may be carried out by dividing the structure into a series of equivalent frames. The moments in these frames may be determined by a computer program or a hand-calculated structural analysis.

This report compares the traditional and new methods of designing flat slabs.

It has already been shown that LUSAS is capable of designing this slab using a finite element analysis.

The slab is now designed by a hand analysis to BS8110. The results for the quantity of reinforcement are quite similar to that provided by LUSAS. However a large discrepancy is observed between the quantities of mid-span reinforcement provided by both methods. BS8110 designates 1300mm/m² while LUSAS requires 950mm/m². This is not to say that LUSAS has under-designed the slab, rather that BS110 adopts a more conservative design approach.

When designing to this slab it is likely cracking of the concrete will occur in the tension zones. This cracking however, does not detract from the safety of the structure provided there is good reinforcement bonding to ensure that the cracks are restrained from opening so that the embedded steel continues to be protected from corrosion.

The Finite Element Method

The finite element method is a general form of analysis to get numerical solutions. It can be applied to stress, heat transfer, fluid flow, electrical fields and more. They are generally applied to complex problems that cannot be analysed by classical or standard methods. The geometry of a problem is broken up into smaller triangular or quadrilateral “elements”. These contain “nodes” in the corners as well as, but not necessarily, along the lines. The nodes are common to the elements it touches so that it must always connect to these elements. This means the geometry of a problem has a continuity of displacement meaning that regardless of what is being analysed (or the magnitude) the elements must remain connected at these points. A mesh is the combination of nodes and elements. The size of the elements and arrangement of the mesh helps to dictate the accuracy of solution. The more elements used and the more regular the mesh the more accurate the solution.

The advantages of the finite element method are that it can be adapted to a wide range of complex problems, it returns results with suitable accuracy for engineers and, using computers, complex problems can be solved relatively quickly. Disadvantages include the difficulties in locating problems that don't allow a computer analysis due to the complexity of a model setup especially in very large scale models, the possible inability to see mistakes that throw off results again to the complexity and the inability of the method to model possible discontinuities of the elements during the analysis.

LUSAS Finite Element Analysis

The analysis was done for the typical flat slab arrangement shown above. It was created in LUSAS civil and structural (academic version). Due to the constraints of the academic version the size of the problem had to be limited. This included the use of a mesh size line division of just four, which reduces the accuracy of the model. Essentially the model analysed is a 12x12x0.2m flat slab supported by one shear wall and six columns spaced 6m apart. The supports are assumed to only support the slab vertically and as such do not represent any monolithic conditions that may be present. The mesh used in the analysis was defined for a thick plate. This was further modelled as an irregular mesh with triangular elements and quadratic interpolation. LUSAS has common structural material properties saved so that a

models material/s can easily be defined. In this case the material attributed to the model was long term C40 concrete as defined by BS8110. These are all defined in the attribute tab.

The loading on the slab is a combination of uniformly distributed dead and live load that gives the worst case scenario for all necessary results. This was achieved by first defining the dead load (DL) as the mass of the concrete (given by the material properties) multiplied by g (9.81 m/s^2). In LUSAS this is known as a body force. The live load (LL) was defined as 5000 N global distributed per unit area i.e. 5 KN/m^2 . The max load, as defined by BS8110, is $1.4\text{DL} + 1.6\text{LL}$ and the min load as 1.0DL . By defining eight separate load cases in LUSAS for the four slab panels each with their two forms of loading as well as the range of max and min for the DL and LL separately it is possible to analyse the worst case scenario for all sixteen possible load combinations. The max and min range of DL being 1.4DL and 1.0DL and the max and min range of LL being 1.6LL and 0.0LL respectfully. A smart load combination is employed with 1.0 as the permanent load factor and 0.4 variable load factor for dead load. The permanent load factor and variable load factor for live load are 1.6 and 0 respectfully. Upon analysis this returns a type of 3D envelope that (for this case) includes bending moment in the X and Y directions in the top and bottom of the slab, the required steel reinforcement in these areas as well as the expected crack widths.

For the steel reinforcement to be calculated a code needs to be chosen to work with (BS8110) as well as bar sizes, spacing, top/bottom cover and steel strength. Other values already decided will have to be repeated for consistency i.e. slabs depth. Changing the bar diameter only affects the steel area calculated by reducing the length of the lever arm but bar diameter and spacing do affect the crack widths found. An iterative process can be used to find the steel actual size and spacing of steel required or, as far more likely, the effect of the change in lever arm will be so small that the chosen bar size and spacing from the results will still be adequate when the reduced lever arm is included.

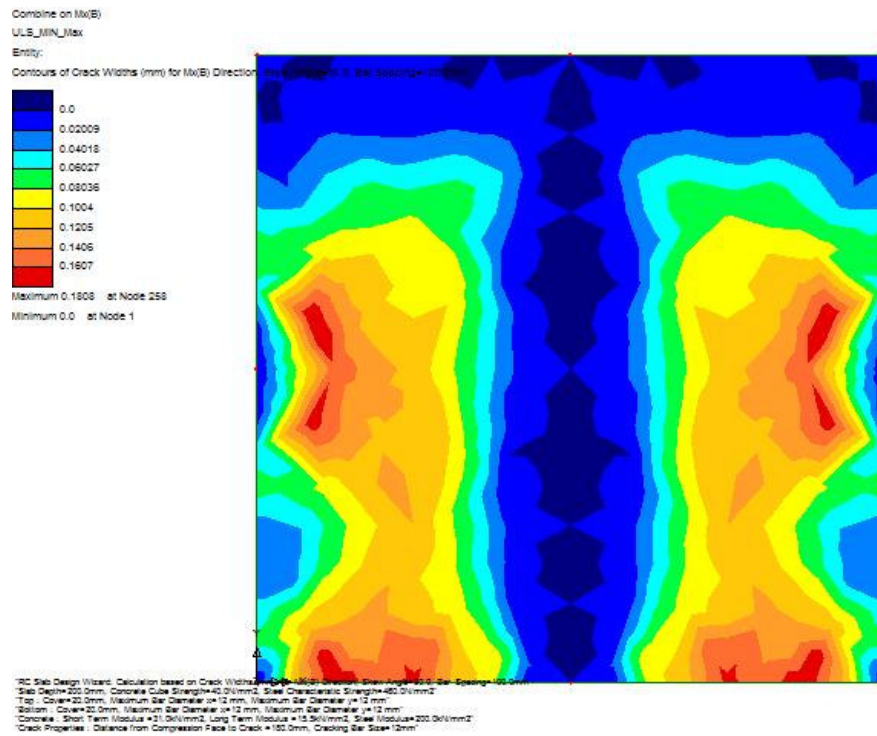
This model arrangement, including slab dimensions, supports, loading etc. can be easily be repeated for any variation. For instance, an altered model that can take account of holes, concrete core supports or a less regular slab shape can easily be defined. LUSAS also has many examples that go through a step-by-step analysis and explain how to perform each command.

Crack Widths

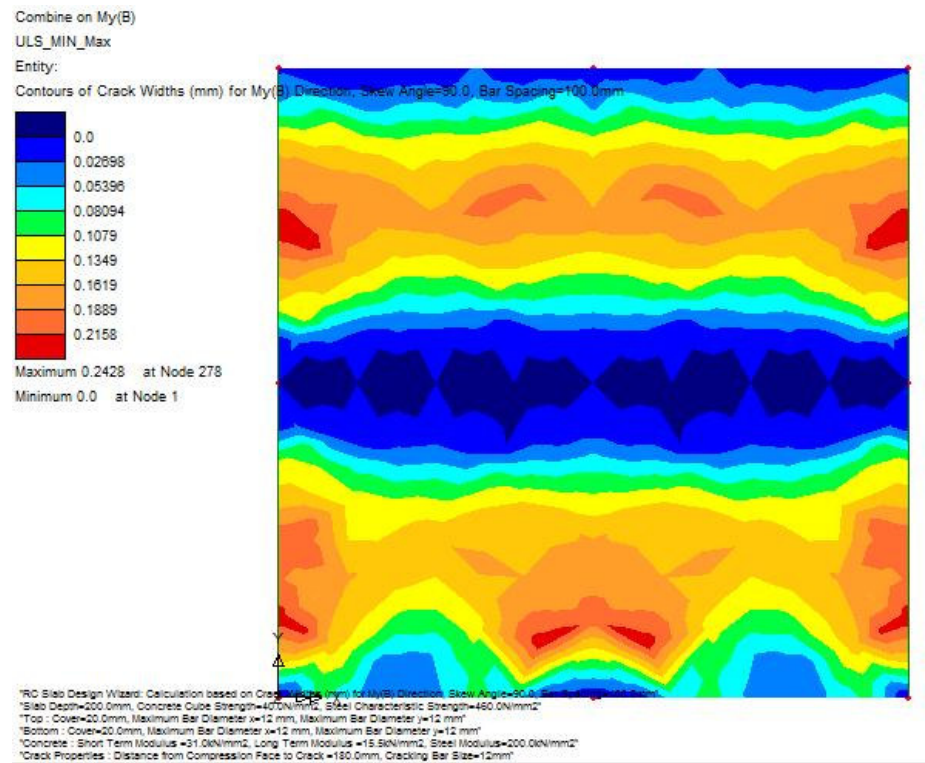
Cracking needs to be controlled in concrete as it is a serviceability requirement. It can, however, affect the strength of reinforced concrete indirectly as it may compromise the cover to the steel reinforcement. This means the steel may corrode under medium to mild conditions. BS8110 defines the max crack width generally as 0.3 mm. As the tensile force is taken to be zero it will not directly affect the calculations though. Finite element analysis cannot take account of this discontinuity of material due to the required compatibility of displacement of the finite elements. LUSAS does not require material properties and thickness to analyse the bending moment in the slab. This is done using the 2D meshed surface with fixed points. Once it has these values an analysis of the given section and bar position can be done to calculate the required steel. Although not shown in LUSAS, it must assume a zero or largely diminished concrete tensile strength otherwise the required balancing steel area calculated would be zero as the concrete in compression would be balanced by the concrete in tension. So although LUSAS requires this continuity it does not affect the required results to a great extent as it is required in the bending moment analysis and not the analysis of the section. Therefore the difference between the two methods in this regard is negligible. The bending moment from BS8110 is found using formulae and any difference should be from this difference in analysis.

The beam size and spacing does affect the magnitude of the crack widths and through the LUSAS analysis it was found that the critical factor for the sizing of the bars in the top of the slab in both directions was crack limitation. This shows that the crack width is a very important factor to be considered in flat slabs.

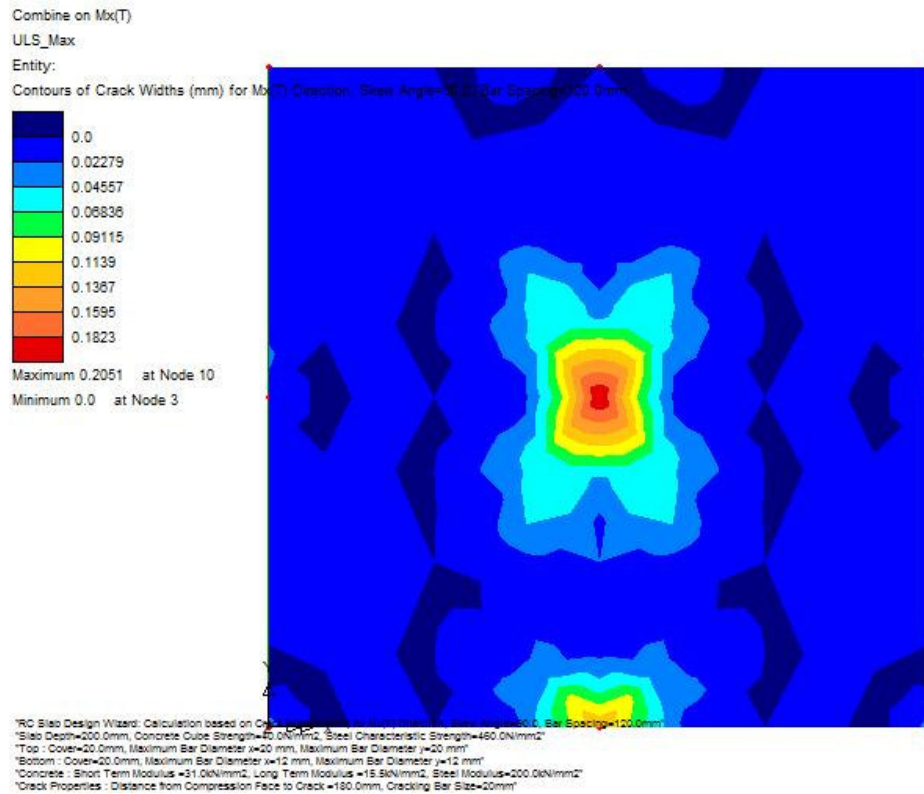
The max crack widths calculated from LUSAS are shown below and show compliance with the 0.3mm max allowed.



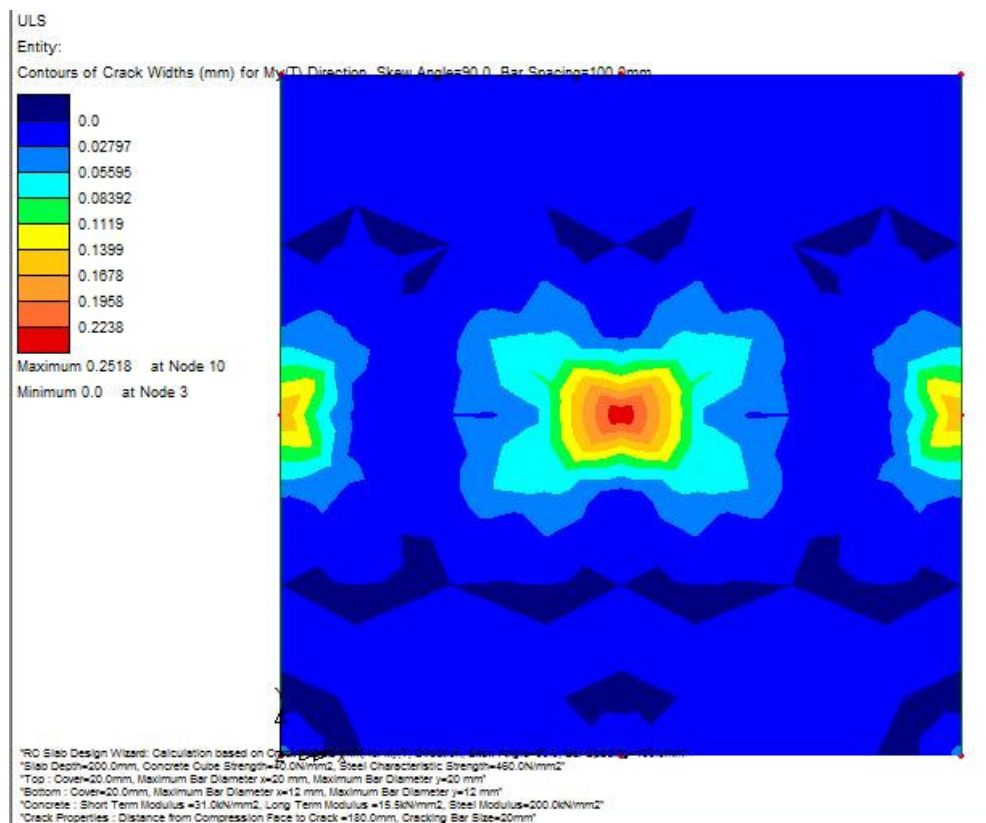
Crack Widths in X direction in bottom of slab (max = 0.181 mm)



Widths in y direction in bottom of slab (max = 0.216 mm)



Widths in x direction in top of slab (max = 0.205 mm)

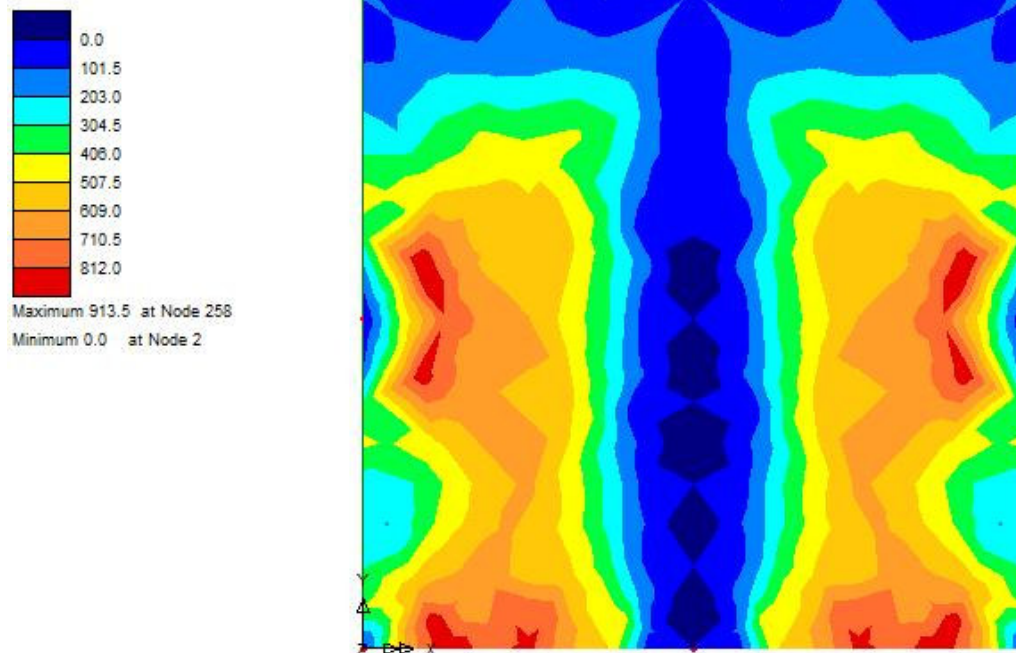


Widths in y direction in top of slab (max = 0.252 mm)

Steel Areas (mm^2/m)

ULS_MIN

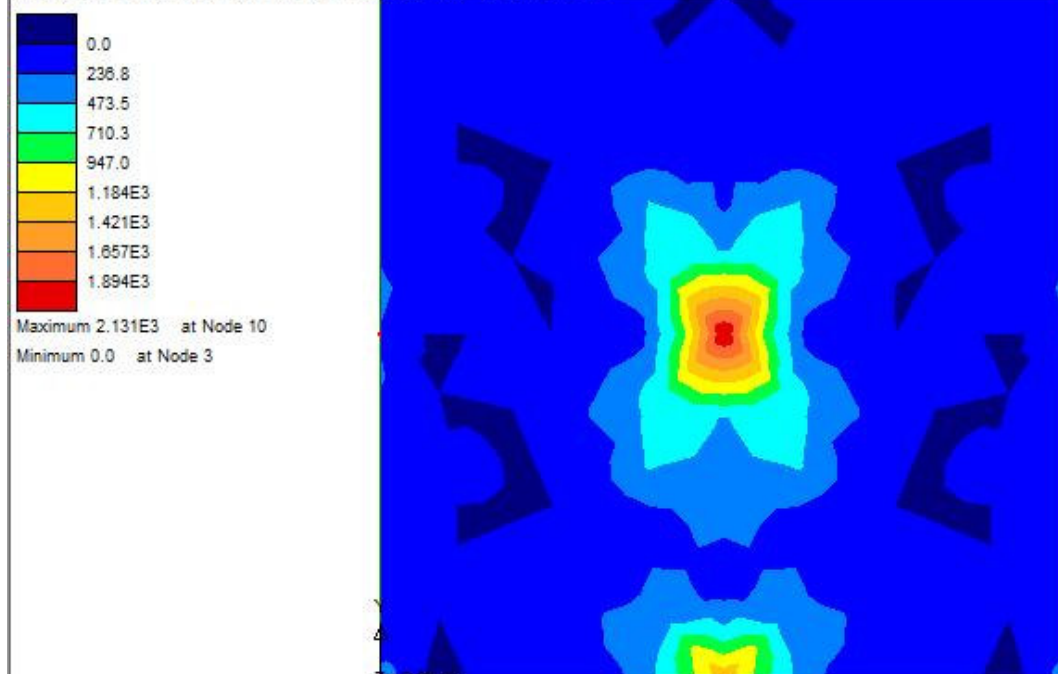
Entity:

Contours of Steel Area (mm^2/m width) for $M_x(B)$ Direction, Skew Angle=90.0

"RC Slab Design Wizard: Calculation based on Steel Area (mm^2/m width) for $M_x(B)$ Direction, Skew Angle=90.0"
 "Slab Depth=200.0mm, Concrete Cube Strength=40.0N/mm², Steel Characteristic Strength=460.0N/mm²"
 "Top : Cover=20.0mm, Maximum Bar Diameter x=12 mm, Maximum Bar Diameter y=12 mm"
 "Bottom : Cover=20.0mm, Maximum Bar Diameter x=12 mm, Maximum Bar Diameter y=12 mm"

ULS

Entity:

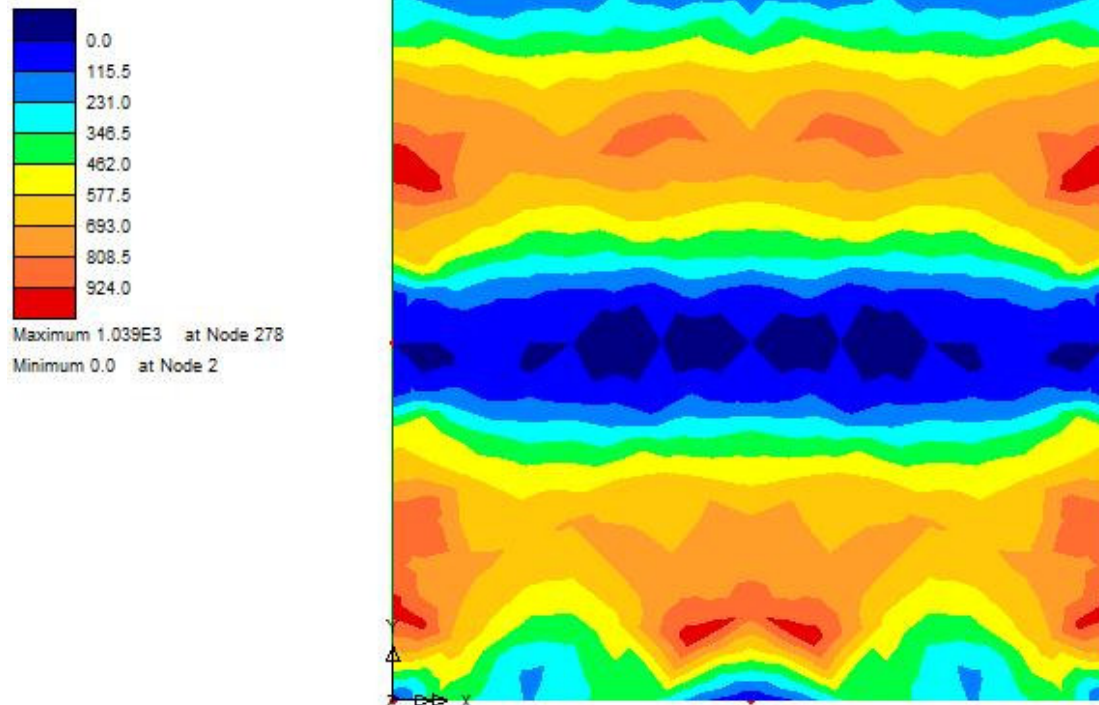
Contours of Steel Area (mm^2/m width) for $M_x(T)$ Direction, Skew Angle=90.0

"RC Slab Design Wizard: Calculation based on Steel Area (mm^2/m width) for $M_x(T)$ Direction, Skew Angle=90.0"
 "Slab Depth=200.0mm, Concrete Cube Strength=40.0N/mm², Steel Characteristic Strength=460.0N/mm²"
 "Top : Cover=20.0mm, Maximum Bar Diameter x=12 mm, Maximum Bar Diameter y=12 mm"
 "Bottom : Cover=20.0mm, Maximum Bar Diameter x=12 mm, Maximum Bar Diameter y=12 mm"

ULS_MIN

Entity:

Contours of Steel Area (mm²/m width) for My(B) Direction, Skew Angle=90.0

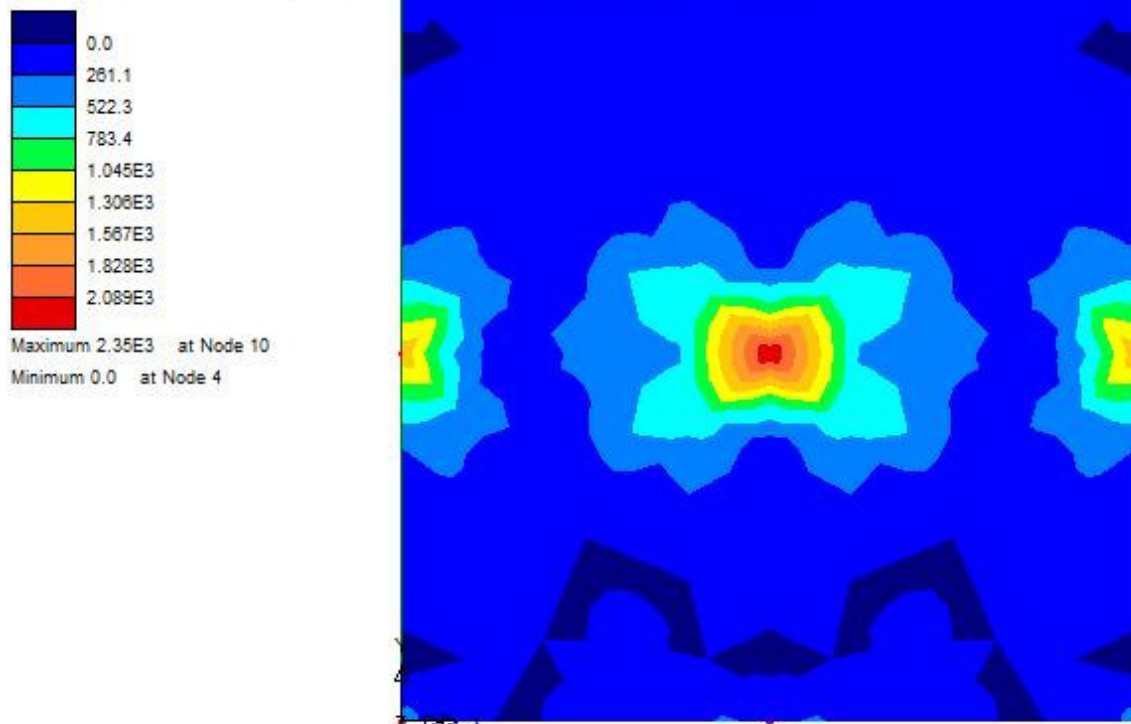


"RC Slab Design Wizard: Calculation based on Steel Area (mm²/m width) for My(B) Direction, Skew Angle=90.0"
 "Slab Depth=200.0mm, Concrete Cube Strength=40.0N/mm², Steel Characteristic Strength=460.0N/mm²"
 "Top : Cover=20.0mm, Maximum Bar Diameter x=12 mm, Maximum Bar Diameter y=12 mm"
 "Bottom : Cover=20.0mm, Maximum Bar Diameter x=12 mm, Maximum Bar Diameter y=12 mm"

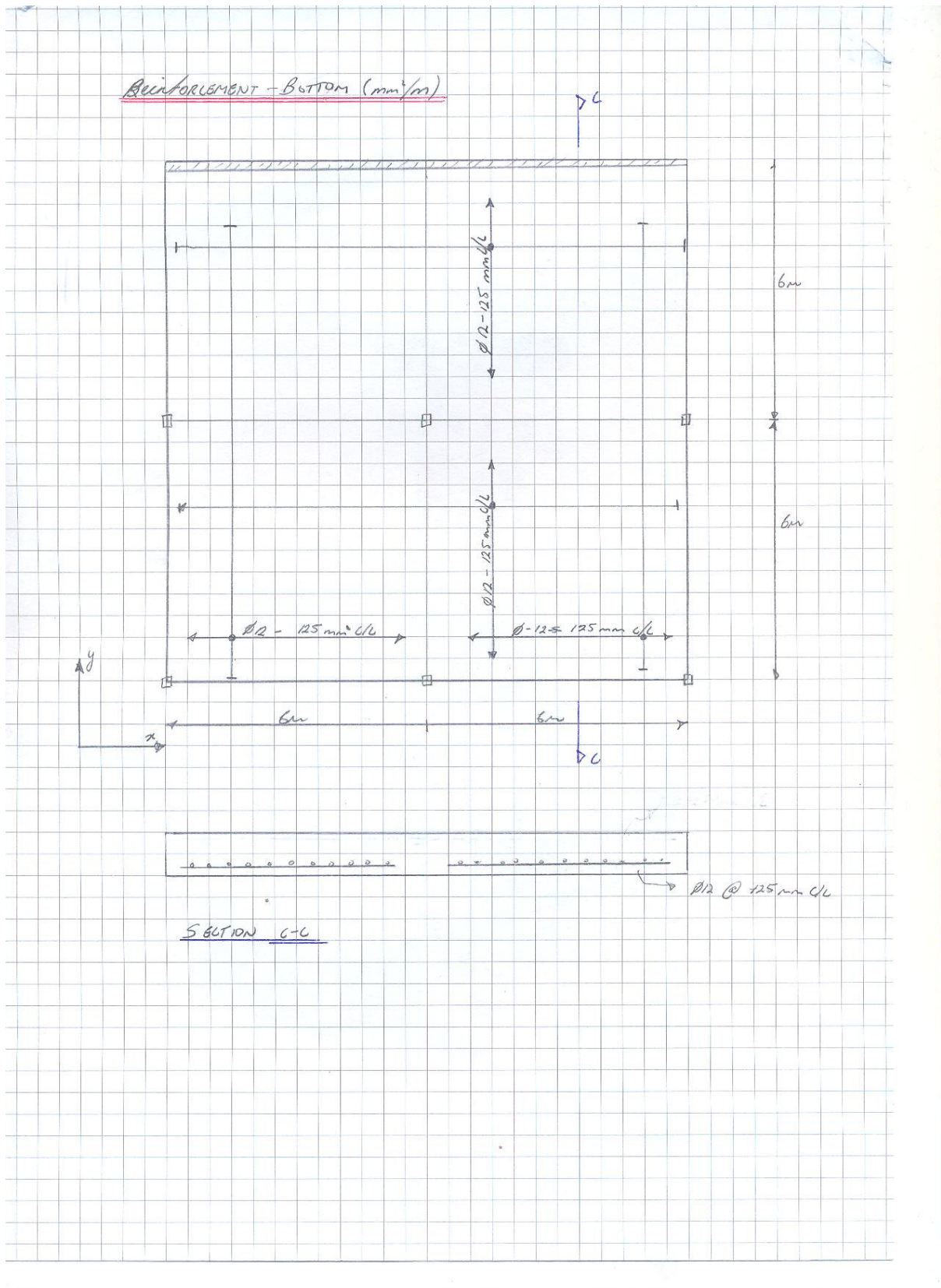
ULS

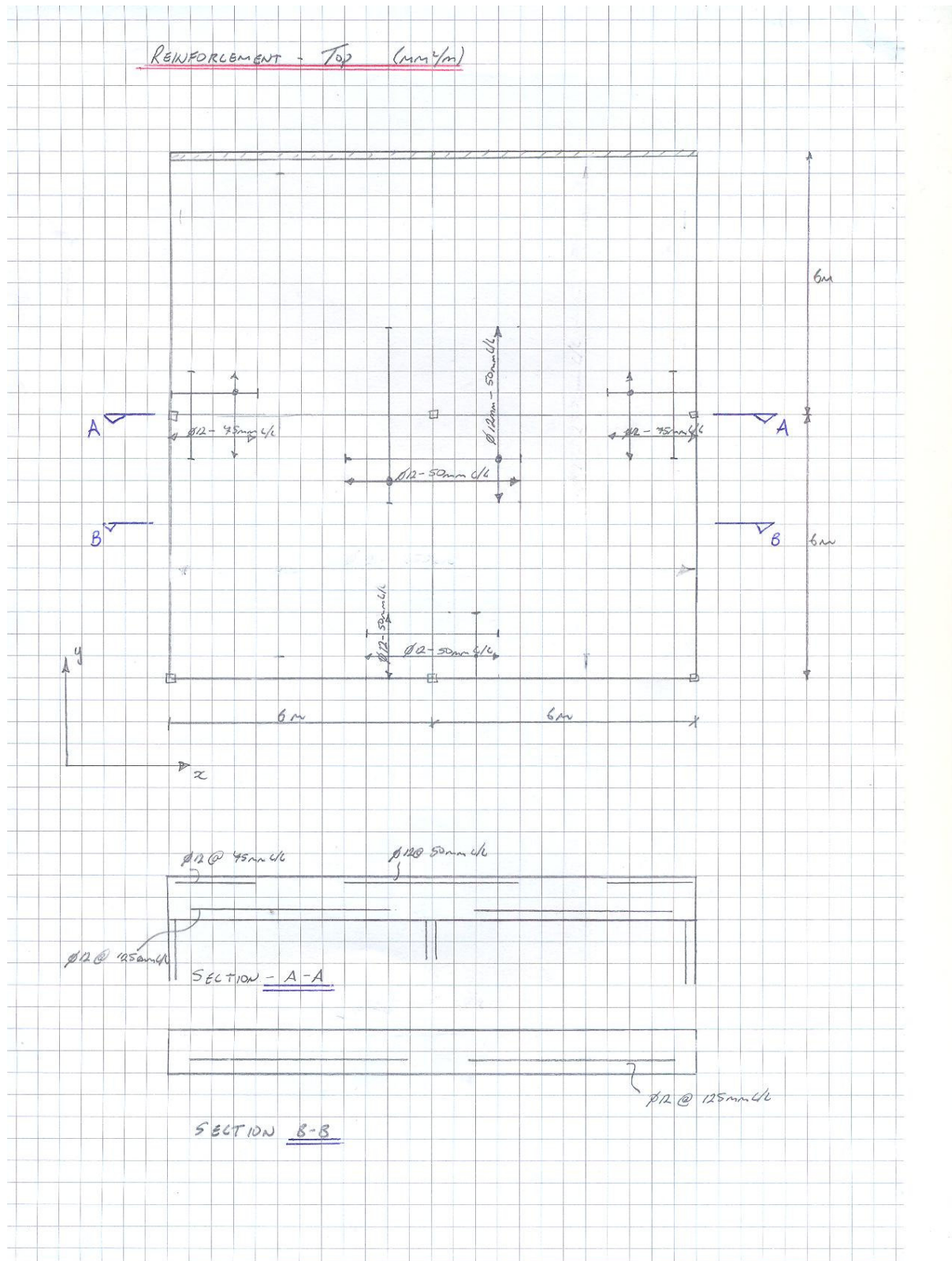
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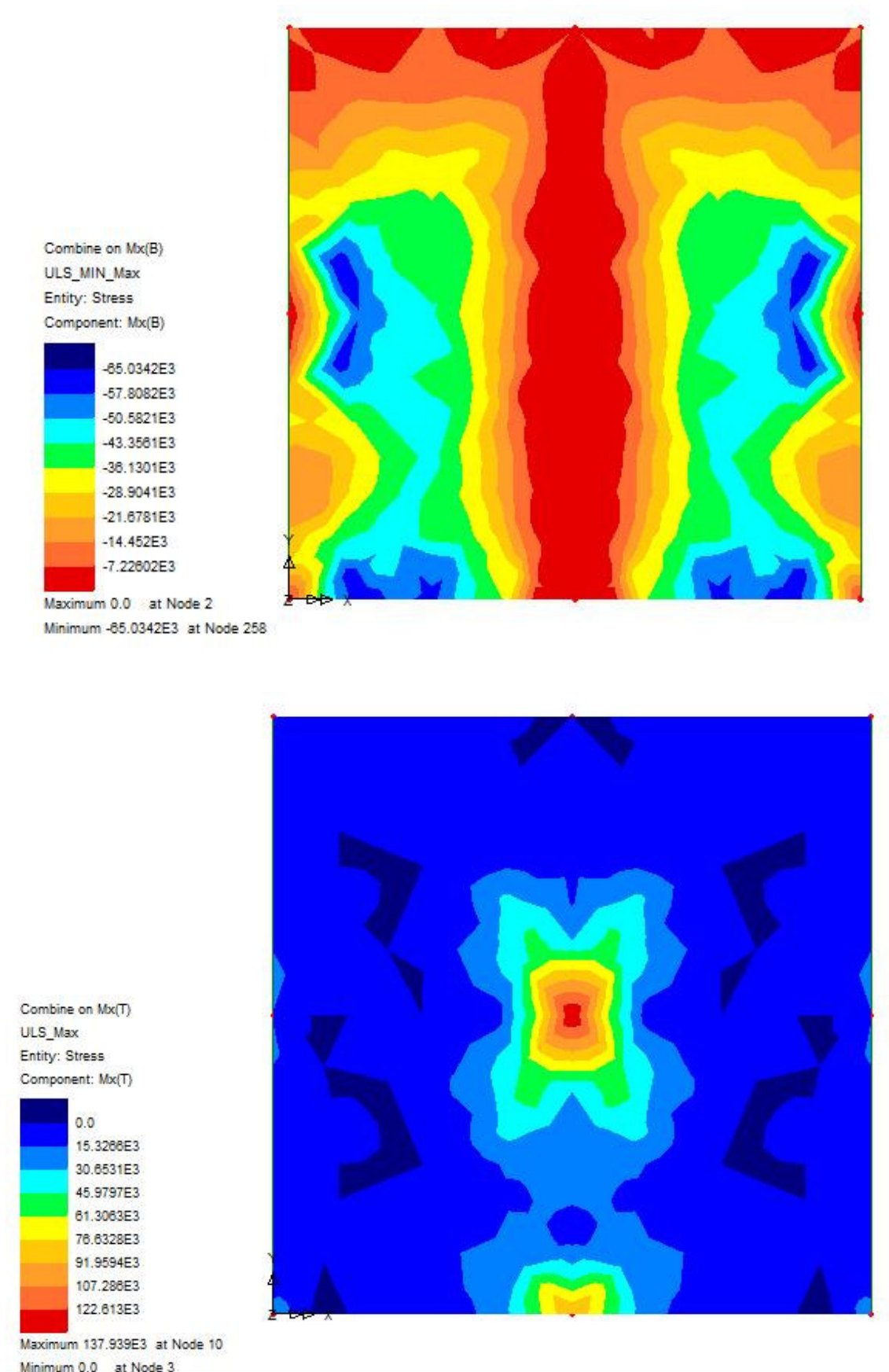
Contours of Steel Area (mm²/m width) for My(T) Direction, Skew Angle=90.0



"RC Slab Design Wizard: Calculation based on Steel Area (mm²/m width) for My(T) Direction, Skew Angle=90.0"
 "Slab Depth=200.0mm, Concrete Cube Strength=40.0N/mm², Steel Characteristic Strength=460.0N/mm²"
 "Top : Cover=20.0mm, Maximum Bar Diameter x=12 mm, Maximum Bar Diameter y=12 mm"
 "Bottom : Cover=20.0mm, Maximum Bar Diameter x=12 mm, Maximum Bar Diameter y=12 mm"



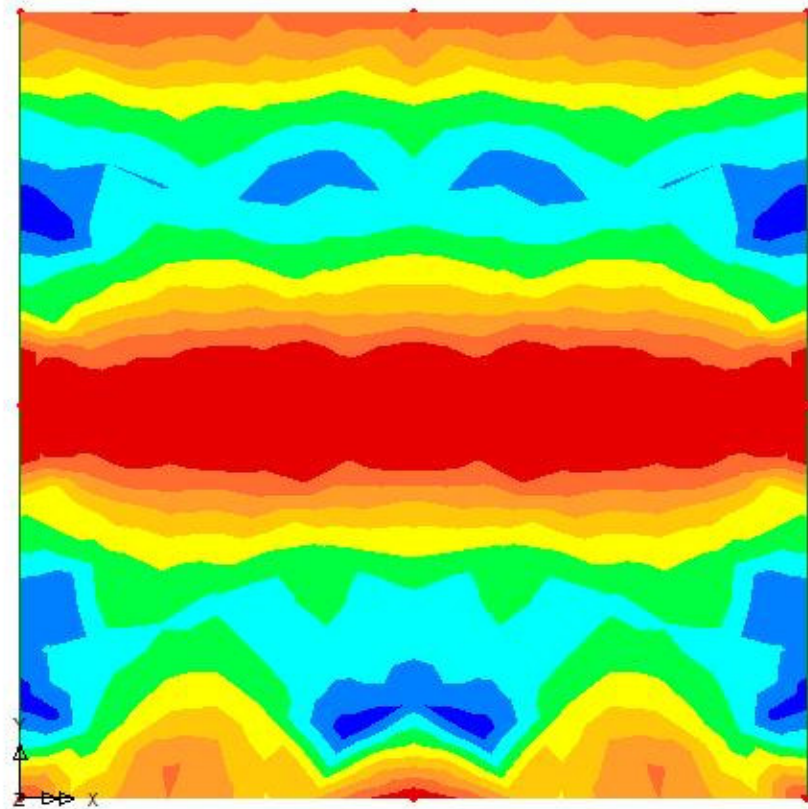


Moments

Combine on My(B)
 ULS_MIN_Max
 Entity: Stress
 Component: My(B)

-67.857E3
-60.3174E3
-52.7777E3
-45.238E3
-37.6983E3
-30.1587E3
-22.619E3
-15.0793E3
-7.53987E3

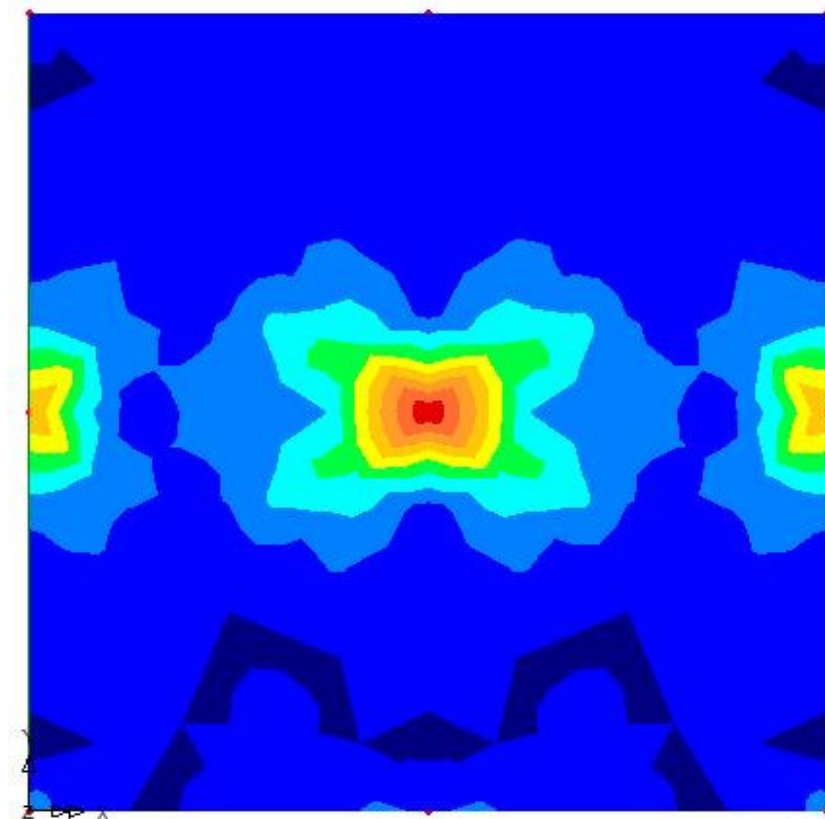
Maximum 0.0 at Node 2
 Minimum -67.857E3 at Node 278



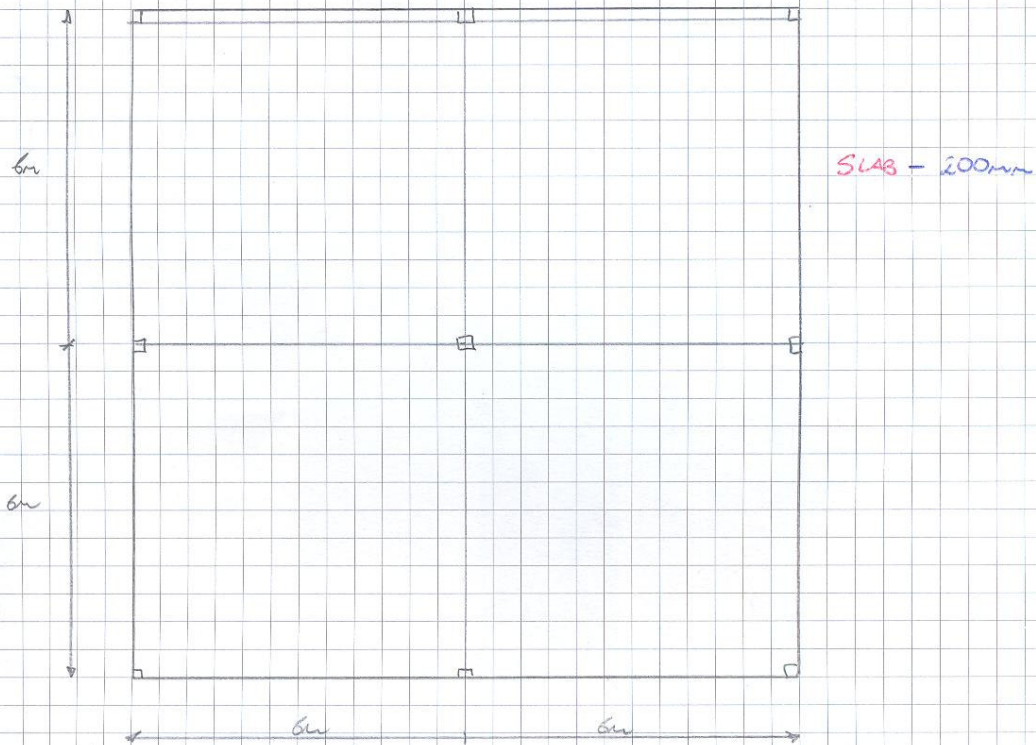
Combine on My(T)
 ULS_Max
 Entity: Stress
 Component: My(T)

0.0
15.2308E3
30.4615E3
45.6923E3
60.9231E3
76.1538E3
91.3846E3
106.615E3
121.846E3

Maximum 137.077E3 at Node 10
 Minimum 0.0 at Node 4



USING THE EMPIRICAL METHOD OF ANALYSIS



THE SLAB ADHERES TO THE CONDITION USE
EMPIRICAL METHOD

CONCRETE	$f_{cu} = 40 \text{ N/mm}^2$
STEEL	$f_y = 460 \text{ N/mm}^2$
COVER	$\approx 20 \text{ mm}$

LOADING

DEAD

$$\text{SELF WEIGHT} \approx (0.2)(24) = 4.8 \text{ kN/m}^2$$

LIVE

$$\approx 5 \text{ kN/m}^2$$

ULTIMATE LOAD

$$\Rightarrow (1.4)(4.8) + 1.6(5) \approx 14.72 \text{ kN/m}^2$$

MOMENTS

MOMENT AT EDGE SAME FOR ALL PANELS

 $F \approx$ ULTIMATE load on PANEL $L \approx$ EFFECTIVE LENGTHMOMENT AT EDGE $\approx -0.02 FL$

$$L \approx 6000 - 125 - 125 \approx 5750 \text{ mm}$$

$$F \approx (14.72)(6)^2 \approx \underline{529.92 \text{ kN}}$$

$$\text{MOMENT @ EDGE} \approx (-0.02)(529.92)(5.7) \approx \underline{-60 \text{ kNm}}$$

$$\text{MOMENT @ CENTRE} \approx (0.083)(529.92)(5.7) \approx \underline{250 \text{ kNm}}$$

$$\text{MOMENT @ INTERIOR SUPPORT} \approx -0.063 FL \approx \underline{-190 \text{ kNm}}$$

$$\text{MOMENT @ CENTRE INTERIOR SPAN} \approx 0.071 FL \approx \underline{214.45 \text{ kNm}}$$

STRIP WIDTHS

$$\text{COLUMNS} \approx (2.5 \times 6) \approx 1.5 \text{ m}$$

$$\text{MIDDLE STRIP} \approx 6 - (2 \times 1.5) \approx 3 \text{ m}$$

POSITIVE MOMENTS

POSITIVE MOMENTS MUST BE DIVIDED 55% TO COLUMN STRIP AND 45% TO MIDDLE STRIP

$$\text{COLUMN STRIP} \sim 250 \times .55 \sim 137.5 \text{ kNm}$$

$$\text{MIDDLE STRIP} \sim 250 \times .45 \sim 112.5 \text{ kNm}$$

AREA OF STEEL REQUIRED FOR POSITIVE MOMENT IN MIDDLE STRIP IN THE EDGE COLUMN STRIP

$$k \sim \frac{M}{b d^2 f_w}$$

$$\sim \frac{137.5 \times 10^5}{(1500)(140)^2(40)} \sim 0.0456$$

$$z \sim d \left[0.5 + \sqrt{0.25 + \frac{0.0456}{0.9}} \right] \sim .95d \sim 1633 \text{ mm}$$

$$A_s \sim \frac{M}{f_y z} \sim \frac{137.5 \times 10^5}{(95)(460)(163.5)} \sim 1924.43 \text{ mm}^2$$

USE T12 @ 75 C/L

NEGATIVE MOMENTS

NEGATIVE MOMENT @ EDGE OF PANEL MUST BE DIVIDED 75% AND 25% TO MIDDLE STRIP

$$\text{COLUMN STRIP} = (60)(.75) = 45 \text{ kNm}$$

$$\text{MIDDLE STRIP} = (60)(.25) = 15 \text{ kNm}$$

AREA OF STEEL REQUIRED FOR NEGATIVE MOMENT IN COLUMN STRIP

$$k = \frac{M}{bd^2f_{cu}} = \frac{45 \times 10^6}{(10^3)(170)^2(40)} = .038$$

$$d = h - \frac{\phi}{2} - c = 200 - \frac{12}{2} - 20 = 174 \text{ mm}$$

$$z = d \left(0.5 + \sqrt{.25 - \frac{k}{0.9}} \right)$$

$$= 174 \left(0.5 + \sqrt{.25 - \frac{.038}{.9}} \right) = 165.3 \text{ mm}$$

$$A_s = \frac{M}{(95)f_y z} = \frac{45 \times 10^6}{(95)(460)(165.3)} = 620 \text{ mm}^2$$

T 12 @ 175 c/c

THIS CASE IN ALL OTHER COLUMNS IN ALL DIRECTIONS.

• TOTAL NEGATIVE MOMENT IN MIDDLE STRIP

$$(-190)(0.25) + 12 = 49.5 \text{ kNm}$$

AREA OF STEEL REQUIRED FOR NEGATIVE MOMENT
IN MIDDLE STRIP

$$k = \frac{M}{bd^2 f_{wy}} = \frac{49.5 \times 10^6}{(1000)(174)^2(40)} = 0.0408$$

$$z = d \left(0.5 + \sqrt{0.25 + \frac{0.0408}{0.9}} \right) = 165.3 \text{ mm}$$

$$A_s = \frac{M}{95 f_y z} = \frac{49.5 \times 10^6}{(95)(460)(165.3)} = 685 \text{ mm}^2$$

USE T12 @ 150 C/C

$A_s \text{ provided} = 754 \text{ mm}^2$

• AREA OF STEEL REQUIRED IN COLUMNS STRIP FOR
NEGATIVE MOMENT AT INTERIOR SUPPORT

$$M = -190 \times 0.45 = 142.5 \text{ kNm}$$

$$k = \frac{M}{bd^2 f_{wy}} = \frac{142.5 \times 10^6}{(1000)(174)^2(40)} = 0.15$$

$$z = d \left[0.5 + \sqrt{0.25 + \frac{0.15}{0.9}} \right] = 0.95d = 165.3$$

$$A_s = \frac{M}{(95 f_y) z} = \frac{142.5 \times 10^6}{(95)(460)(165.3)} = 2214 \text{ mm}^2$$

T12 @ 50 C/C

• AREA OF STEEL REQUIRED IN COLUMN STRIP FOR POSITIVE MOMENT IN MIDDLE SPAN

$$M = (214.45)(55) = 117.94 \text{ kNm}$$

$$k = \frac{M}{bd^2 f_{lu}} = \frac{117.94 \times 10^6}{(3000)(174)^2(40)} = 0.032$$

$$z = d \left(0.5 + \sqrt{0.25 - \frac{k}{.9}} \right) = .95d = 165.3 \text{ mm}$$

$$A_s = \frac{M}{.95 f_y z} = \frac{117.94 \times 10^6}{.95(460)(165.3)} = 1632.7 \text{ mm}^2$$

USE T12 @ ~~150~~ 4/c

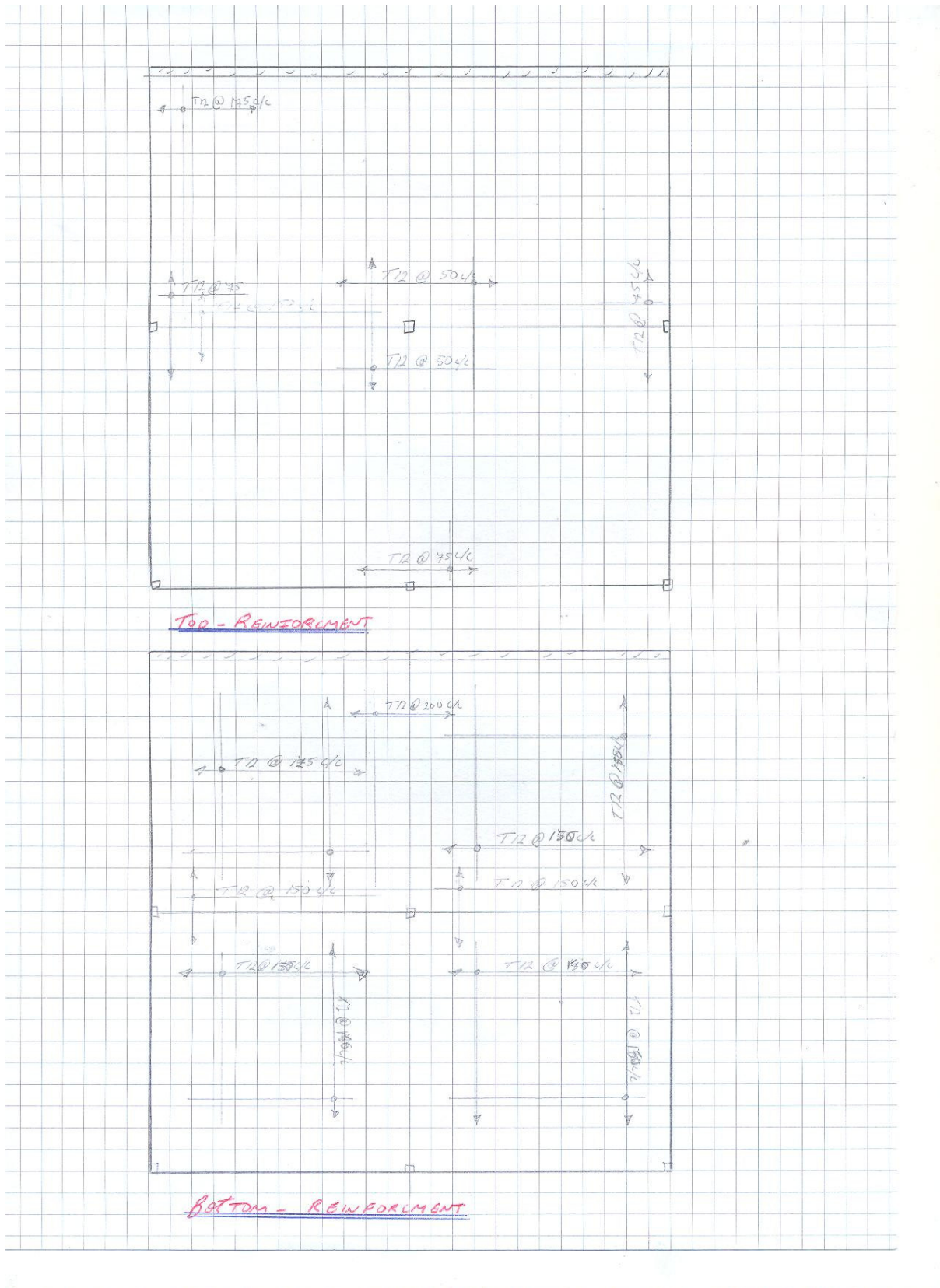
• AREA OF STEEL REQUIRED FOR POSITIVE MOMENT IN THE MIDDLE STRIP

$$k = \frac{M}{bd^2 f_{lu}} = \frac{112.5 \times 10^6}{(3000)(174)^2(40)} = 0.030$$

$$z = d \left(0.5 + \sqrt{0.25 - \frac{0.030}{.95}} \right) = .95d$$

$$A_s = \frac{M}{0.95 f_y z} = \frac{112.5 \times 10^6}{(0.95)(460)(165.3)} = 1554.4 \text{ mm}^2$$

USE T12 @ ~~150~~ 4/c



References

- Concepts and Applications of Finite Element Analysis, 2nd Ed., Robert D. Cook, John Wiley and Sons 1981
- Reinforced Concrete Design, 5th Ed. To BS8110, Mosley, Bungey and Hulse, J. W. Arrowhead Ltd. 1999
- BS 8110